

Energy extraction from extremal charged black holes due to the BSW effect

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Two particles can collide in the vicinity of a rotating black hole producing the divergent energy in the centre of mass frame (the BSW effect). However, it was shown recently that an observer at infinity can register quite modest energies E and masses m which obey some upper bounds. In the present work the counterpart of the original BSW effect is considered that may occur even for radial motion of colliding particles near charged static black holes. It is shown that in some scenarios there are no upper bound on E and m . Thus the high-energetic and superheavy products of the BSW effect in this situation are, in principle, detectable at infinity.

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The investigation of high-energetic particles collisions in the vicinity of rotating black holes was initiated in [1] where the collisional version of the Penrose process [2] was investigated. The new urge to considering such processes came from an interesting observation made in Ref. [3]. It was found there that two particles which move towards the horizon of the extremal black holes can produce an infinity energy in the centre of mass frame $E_{c.m.}$. This effect (called the BSW one after the names of its authors) provoked a large series of works and is under active study currently. The most part of them was restricted to the investigation of the vicinity of the horizon where collision occurs. Meanwhile, of special interest is the question whether the consequences of this effect can be detected (at least, in principle) in a laboratory. In other words, can an observer at infinity register ultra-high energy and/or superheavy particles? In [4], [5] the phenomenological description of fluxes from emergent particles due to collisions was suggested. The recent works [6], [7], [8] discussed

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the collisional process when one of two created particles escapes to infinity. It turned out that in spite of divergent $E_{c.m.}$, the energy and mass which can be registered at infinity as a result of the BSW effect are rather modest. In this sense, the results of [6], [7] and [8] are somewhat disappointing in the observational sense (although they do allow some indirect imprints of the BSW effect on its products measured at infinity).

Meanwhile, there exists also the counterpart of the original BSW effect that arises not due to rapid rotation but due to the charge of a black hole [9]. It reveals itself even for a pure radial motion. From another hand, it is known that for the RN black hole the analogue of the Penrose process [2] also exists [10] (details of a more general case when a black hole is both rotating and charged can be found in the review [11]). In the present work we consider the collisional version of the Penrose process for charged black holes and show that, contrary to the situation with rotating black holes, the significant energy extraction is possible. Without pretending to direct applications in realistic astrophysics, the example with a charged nonrotating black hole can be viewed as a useful model that shows how the BSW effect can have direct manifestations detectable at infinity.

Let us consider the spherically symmetric metric

$$ds^2 = -N^2 dt^2 + \frac{dr^2}{N^2} + r^2(d\theta^2 + d\phi^2 \sin^2 \theta) \quad (1)$$

describing the extremal Reissner-Nordström metric (RN). Then, the lapse function

$$N = 1 - \frac{r_+}{r} \quad (2)$$

where r_+ is the horizon radius.

Let two particles 1 and 2 with masses m_1 and m_2 be injected from infinity, collide and produce two new particles 3 and 4 with masses m_3 and m_4 . We restrict ourselves by pure radial motion only (generalization to nonzero angular momenta is straightforward). Then, the conservation laws of the electric charge, energy and radial momentum give us

$$q_1 + q_2 = q_3 + q_4, \quad (3)$$

$$X_1 + X_2 = X_3 + X_4, \quad (4)$$

$$\varepsilon_1 Z_1 + \varepsilon_2 Z_2 = \varepsilon_3 Z_3 + \varepsilon_4 Z_4. \quad (5)$$

Here, $X = E - q\varphi$ where φ is the electric potential. $Z_i = \sqrt{X_i^2 - m_i^2 N^2}$. As usual, we assume that $X > 0$ outside the horizon (the forward in time condition) but $X_H = 0$ is

possible on the horizon. Hereafter, subscript "H" means that the corresponding quantity is calculated on the horizon. It is implied that $\varepsilon_i = -1$ if a particle labeled by subscript "i" moves towards the horizon and $\varepsilon_i = +1$ if it moves outwardly. For the extremal RN metric, $\varphi = \frac{r_+}{r} = 1 - N$ (for definiteness, the electric charge of a black hole $Q = r_+ > 0$).

The BSW effect occurs if particle (say) 1 is critical and particle 2 is usual [9]. It means, by definition, that $(X_1)_H = 0$, so $E_1 = q_1$ and $(X_2)_H \neq 0$. Further consideration goes very closely to [8] but gives the results qualitatively different from those for rotating uncharged black holes [6], [7], [8].

We must have $Z^2 = X^2 - m^2 N^2 \geq 0$, the zeros of Z give us the turning points. The condition $Z = 0$ can be rewritten as $X = mN$, whence

$$q \leq q_0 = \frac{E - mN}{1 - N}. \quad (6)$$

On the horizon, $q_0 = q_H \equiv E$. Hereafter, it is assumed that $q > 0$ for a critical (or near-critical) particle, otherwise one cannot achieve the condition of criticality $X_H = 0$.

Near the horizon, (6) turn into

$$q_0 = E + N(E - m) + O(N^2). \quad (7)$$

We will be interested in the situation when a particle (denoted as particle 3) escapes to infinity from the immediate vicinity of the horizon. This is possible in 2 cases.

a) $E_3 \geq m_3$, $q_3 < q_H$, $\varepsilon_3 = +1$. The act of collision occurs in the allowed region just near the horizon, afterwards particle 3 escapes to infinity.

b) $E_3 \geq m_3$, $q_H < q_3 < q_0$, $\varepsilon_3 = +1$ or $\varepsilon_3 = -1$. Collision occurs just outside the potential barrier near the horizon, particle 3 is slightly noncritical. The condition $\varepsilon_3 = -1$ means in this context that particle 3 is moving inwardly, approaches the turning point and bounces back in the outward direction. We consider all these types of scenarios in the vicinity of the horizon where $N \ll 1$.

It is convenient to write $q = E(1 + \delta)$. Then, in case (a) $\delta < 0$. In case (b) $\delta \geq 0$ but it is bounded from the above. Indeed, forward in time condition $X > 0$ gives us

$$\delta < \frac{N}{1 - N} \quad (8)$$

The condition $q < q_0$ entails

$$\delta < N(1 - \frac{m}{E}) \quad (9)$$

which is more tight than (9).

In the near-horizon region, the lapse function N is a small quantity. For what follows, we need also the expansions for the quantity Z . This can be found separately for different kinds of particles.

1) Usual particle. For such a particle, $X_H \neq 0$, so we obtain

$$Z = X + O(N^3) \quad (10)$$

where

$$X = X_H + qN, \quad X_H = E - q. \quad (11)$$

2) Critical particle. Now, $X_H = 0$, $q = E$, so

$$X = EN, \quad (12)$$

$$Z = N\sqrt{E^2 - m^2}. \quad (13)$$

3) Near-critical particle

Let us consider a particle which is not exactly critical but, rather, near-critical. For such a particle, $\delta \ll 1$. We can adjust the value δ to the small N choosing it in the form of series

$$\delta = C_1 N + C_2 N^2 + \dots \quad (14)$$

Then,

$$X = EN(1 - C_1) + O(N^2), \quad (15)$$

$$Z = N\sqrt{E^2(1 - C_1)^2 - m^2} + O(N^2) \quad (16)$$

Now, we can apply the near-horizon expansion to different scenarios of escaping. In case (a), the condition $\delta < 0$ and eq. (14) give us

$$C_1 < 0. \quad (17)$$

In case (b), we must take into account the presence of the turning point outside the horizon. Then, it follows from (9) that

$$0 \leq C_1 \leq (C_1)_m = 1 - \frac{m}{E}. \quad (18)$$

The scenario in which a near-critical particle has $\varepsilon_3 = -1$ immediately after collision and thus moves inwardly, we will call IN scenario for shortness. If after collision $\varepsilon_3 = +1$ we will

call it "OUT" scenario. In turn, we will add superscript "-" if $\delta < 0$ and "+" if $\delta \geq 0$. In other words, we enumerate possible types of scenarios characterizing them by signs of two quantities - ε and δ . In general, there are 4 combinations: OUT-, OUT+, IN+, and IN-. However, the scenario IN- should be rejected since it corresponds to a particle 3 falling down in a black hole whereas we want it to escape to infinity..

In any scenario, particle 4 is usual and falls into a black hole ($\varepsilon_4 = -1$). This is obtained in [8] by analyzing eqs. (4), (5) for the Kerr case but actually it is insensitive to the form of the metric and relies on pure algebra, so it applies also to dirty rotating black holes [7] and to the RN one.

Using (5) with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = -1$ we have $-Z_1 - Z_2 = -Z_4 + \varepsilon_3 Z_3$. Then, using (10), (12), (16) we obtain

$$F \equiv A + E_3(C_1 - 1) = \varepsilon_3 \sqrt{E_3^2(1 - C_1)^2 - m_3^2} \quad (19)$$

where

$$A \equiv E_1 - \sqrt{E_1^2 - m_1^2}, \quad (20)$$

$$0 \leq A \leq E_1. \quad (21)$$

Taking the square of (19) we have

$$C_1 = 1 - \frac{m_3^2 + A^2}{2E_3A}, \quad (22)$$

$$F = \frac{A^2 - m_3^2}{2A}. \quad (23)$$

It is seen from (22) that eq. (18) is satisfied automatically. According to (19), $sign F = \varepsilon_3$.

Now we will discuss different scenarios separately.

OUT- $\varepsilon_3 = +1$, $C_1 < 0$

It follows from (22) and (23) that

$$m_3 \leq E_3 \leq \mu = \frac{m_3^2 + A^2}{2A}. \quad (24)$$

$$m_3 \leq A, \quad (25)$$

$$\mu \leq A \leq E_1. \quad (26)$$

There is no energy extraction in this case. In particular, if $E_1 = E_2 = m$, the efficiency of extraction $\eta = \frac{E_3}{E_1 + E_2} \leq \frac{1}{2}$. Moreover, the quantity μ is a monotonically decreasing function

of E_1 . Scenario OUT− was considered in the 1st version of preprint [7] but two other scenarios which are the most interesting were overlooked there. It is their consideration which we now turn to.

OUT+ $\varepsilon_3 = +1, C_1 \geq 0$

Then, it follows from (22) that

$$E_3 \geq \lambda \equiv \frac{m_3^2 + A^2}{2A} \quad (27)$$

and it follows from (23) that (25) holds. Thus there is an upper bound on m_3 but there is no such a bound on E_3 , so extraction of energy exists and is not restricted! (The reservation is in order that we work in the test particle approximation, so we neglect backreaction of particles on the black hole metric). Moreover, we have a lower bound (27) instead of the upper one typical of the rotating black hole case [6], [7], [8]. When $m_1 \rightarrow 0$, $A \rightarrow 0$ and $\lambda \rightarrow \infty$.

IN+ $\varepsilon_3 = -1, C_1 \geq 0$.

Then, $F \leq 0$, so we have from (23) that

$$m_3 \geq A. \quad (28)$$

The conclusion about the lower bound (27) obtained from $C_1 \geq 0$ now applies as well.

Thus the scenario OUT− gives no energy extraction and, apart from this, it forbids creation of superheavy particles. Scenario OUT+ allows ultra-high energetic particles but with the upper bound on their mass. The most interesting scenario is IN + since it predicts unbound energies and superheavy particles with the lower (not upper!) bound on the mass. Moreover, it is detection of superheavy particles that enables to distinguish between the result of a "standard" Penrose process and its collisional version in combination with the BSW effect. This is in sharp contrast with the case of the Kerr metric [6], [8] and more general dirty rotating black holes [7]. Thus we have two quite different situations for the rotating and static charged black hole in what concerns the combination of the BSW effect and collisional Penrose process.

One reservation is in order. Infalling particle 1 is critical and particle 3 observed at infinity is near-critical. Therefore, if $E_3 \gg E_1$ it means that simultaneously $q_3 \gg q_1$ (if, instead, all charges have the same order we return to the situation discussed in [9] when there is no significant energy extraction). This requires deeply inelastic collision with participation

of composite particles, in which case it is natural to assume that $q_3/q_1 < 137$ as usual. Meanwhile, we would like to stress that the main result of our work consists in the fact that enhancement of the observable energy due to the BSW effect is possible in principle, so this can motivate search for more realistic circumstances when extraction of energy due to the BSW effect can occur. In particular, it is interesting to consider the possibility of the collisional Penrose process for the Kerr-Newman metric that combines both opposite limiting cases of the BSW effect (due to the black hole charge and due to rotation) [12].

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